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1.

$$g(x) = \frac{6x + 12}{x^2 + 3x + 2} - 2, \quad x \geq 0$$

(a) Show that $g(x) = \frac{4 - 2x}{x + 1}, \quad x \geq 0$

(3)

(b)

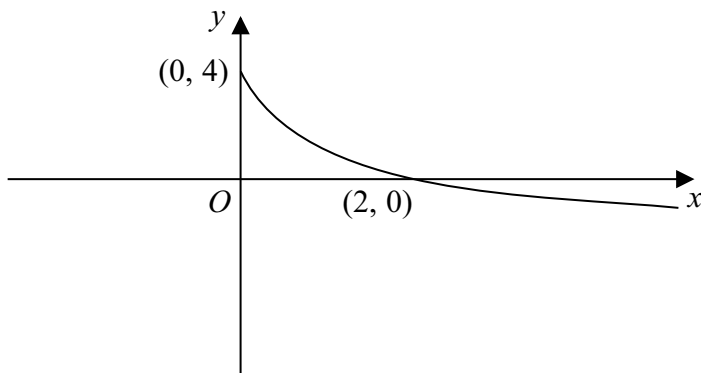


Figure 1

Figure 1 shows a sketch of the curve with equation $y = g(x), \quad x \geq 0$

The curve meets the y -axis at $(0, 4)$ and crosses the x -axis at $(2, 0)$.

On separate diagrams sketch the graph with equation

(i) $y = 2g(2x),$

(ii) $y = g^{-1}(x).$

Show on each sketch the coordinates of each point at which the graph meets or crosses the axes.

(5)



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3.

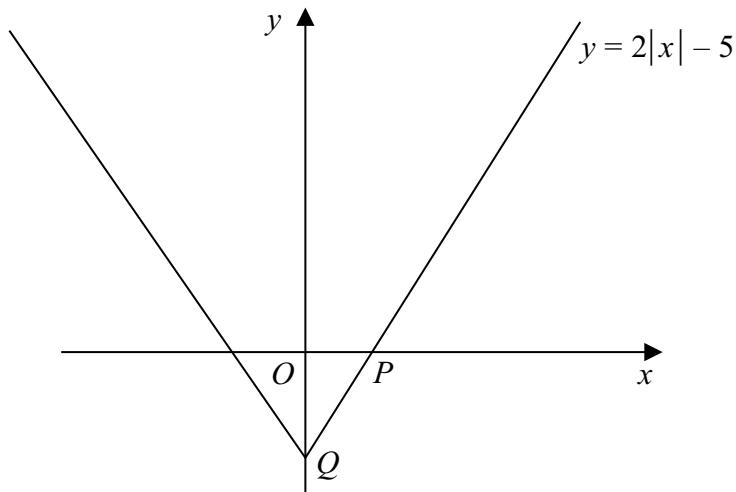


Figure 2

Figure 2 shows a sketch of the graph with equation $y = 2|x| - 5$.

The graph intersects the positive x -axis at the point P and the negative y -axis at the point Q .

(a) State the coordinates of P and the coordinates of Q . (2)

(b) Solve the equation $2|x| - 5 = 3 - x$ (3)



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4. (a) On the same diagram, sketch and clearly label the graphs with equations

$$y = e^x \quad \text{and} \quad y = 10 - x$$

Show on your sketch the coordinates of each point at which the graphs cut the axes. (3)

- (b) Explain why the equation $e^x - 10 + x = 0$ has only one solution. (1)

- (c) Show that the solution of the equation

$$e^x - 10 + x = 0$$

lies between $x = 2$ and $x = 3$ (2)

- (d) Use the iterative formula

$$x_{n+1} = \ln(10 - x_n), \quad x_1 = 2$$

to calculate the values of x_2 , x_3 and x_4 .

Give your answers to 4 decimal places. (3)



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Question 4 continued



5. (i) (a) Show that $\frac{d}{dx}\left(x^{\frac{1}{2}} \ln x\right) = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$

(3)

The curve with equation $y = x^{\frac{1}{2}} \ln x$, $x > 0$ has one turning point at the point P .

(b) Find the exact coordinates of P . Give your answer in its simplest form.

(4)

(ii) A curve C has equation $y = \frac{x-k}{x+k}$, where k is a positive constant.

Find $\frac{dy}{dx}$, and show that C has no turning points.

(4)



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6.

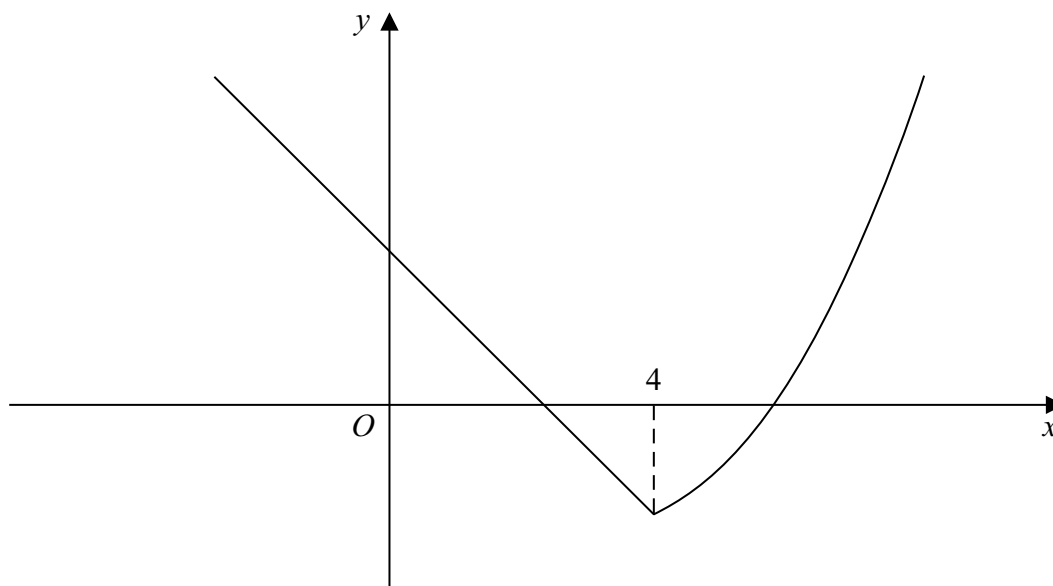


Figure 3

Figure 3 shows a sketch of the graph of $y = f(x)$ where

$$f(x) = \begin{cases} 5 - 2x, & x \leq 4 \\ e^{2x-8} - 4, & x > 4 \end{cases}$$

(a) State the range of $f(x)$. (1)

(b) Determine the exact value of $ff(0)$. (2)

(c) Solve $f(x) = 21$
 Give each answer as an exact answer. (5)

(d) Explain why the function f does not have an inverse. (1)



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8. (a) Express $9 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α to 4 decimal places.

(3)

- (b) (i) State the maximum value of $9 \cos \theta - 2 \sin \theta$

(ii) Find the value of θ , for $0 < \theta < 2\pi$, at which this maximum occurs.

(3)

Ruth models the height H above the ground of a passenger on a Ferris wheel by the equation

$$H = 10 - 9 \cos\left(\frac{\pi t}{5}\right) + 2 \sin\left(\frac{\pi t}{5}\right)$$

where H is measured in metres and t is the time in minutes after the wheel starts turning.



- (c) Calculate the maximum value of H predicted by this model, and the value of t , when this maximum first occurs. Give your answers to 2 decimal places.

(4)

- (d) Determine the time for the Ferris wheel to complete two revolutions.

(2)



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9.

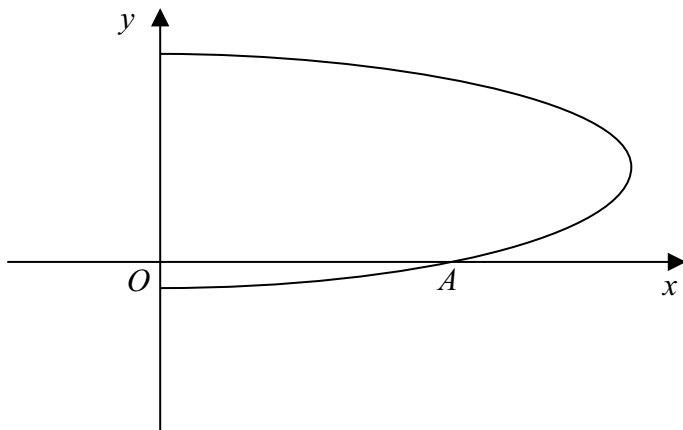


Figure 4

Figure 4 shows a sketch of the curve with equation $x = (9 + 16y - 2y^2)^{\frac{1}{2}}$.

The curve crosses the x -axis at the point A .

- (a) State the coordinates of A . (1)

- (b) Find an expression for $\frac{dx}{dy}$, in terms of y . (3)

- (c) Find an equation of the tangent to the curve at A . (4)



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Question 9 continued

Lined area for student response to Question 9.

(Total 8 marks)

Q9

TOTAL FOR PAPER: 75 MARKS

END

